2,3

16 $r: \begin{cases} x_1 - 2x_2 + 3x_3 = 1 \\ x_1 + x_2 - x_3 = -1 \end{cases}$ $\qquad R_1 = \{O; e_1, e_2, e_3\}$

$R' = \{q = O + 3e_1,\ u_1 = e_1 - e_2,\ u_2 = e_2 - e_3,\ u_3 = e_1 + e_3\}$

$$C(R', R_1) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \longrightarrow \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{pmatrix}}_{P \text{ en } R_1} = \underbrace{\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}_{C(R', R_1)} \cdot \underbrace{\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ 1 \end{pmatrix}}_{P \text{ en } R'}$$

$$\begin{cases} x_1 = x_1' + x_3' + 3 \\ x_2 = -x_1' + x_2' \\ x_3 = -x_2' + x_3' \end{cases} \xrightarrow{r} \begin{cases} x_1' + x_3' + 3 - 2(-x_1' + x_2') + 3(-x_2' + x_3') = 1 \\ x_1' + x_3' + 3 - x_1' + x_2' + x_2' - x_3' = -1 \end{cases} \Longrightarrow$$

$$\Longrightarrow \boxed{\begin{cases} 3x_1' - 5x_2' + 4x_3' = -2 \\ 2x_2' = -4 \end{cases} \equiv r'}$$

17 $A = (1,0,0,0)$ $\quad B = (1,1,1,1)$ $\quad H \equiv x_1 - x_2 + x_3 - x_4 = 2$ $\quad \pi = (1,0,2,-1) + L(\{(1,1,1,0), (1,1,0,1)\})$

$R = \{(1,-1,0,0); (1,1,0,0), (0,1,1,0), (0,0,1,1), (0,0,0,1)\}$

Primero hacemos el cambio de referencia:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ 1 \end{pmatrix} = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right) \cdot \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x_1 = x_1' + 1 \\ x_2 = x_1' + x_2' - 1 \\ x_3 = x_2' + x_3' \\ x_4 = x_3' + x_4' \end{cases}$$

$$A = (1,0,0,0) \longrightarrow \begin{cases} 1 = x_1' + 1 \\ 0 = x_1' + x_2' - 1 \\ 0 = x_2' + x_3' \\ 0 = x_3' + x_4' \end{cases} \Longrightarrow \begin{cases} x_1' = 0 \\ x_2' = 1 \\ x_3' = -1 \\ x_4' = 1 \end{cases} \qquad \boxed{A' = (0, 1, -1, 1)}$$

$$B = (1,1,1,1) \longrightarrow \begin{cases} 1 = x_1' + 1 \\ 1 = x_1' + x_2' - 1 \\ 1 = x_2' + x_3' \\ 1 = x_3' + x_4' \end{cases} \Longrightarrow \begin{cases} x_1' = 0 \\ x_2' = 2 \\ x_3' = -1 \\ x_4' = 2 \end{cases} \qquad \boxed{B' = (0, 2, -1, 2)}$$

$$H \to \{ x_1' + 1 - x_1' - x_2' + 1 + x_2' + x_3' - x_3' - x_4' = 2 \Longrightarrow \boxed{H' \equiv x_4' = 0}$$

$$\pi \to \begin{cases} x_1 = 1 + \lambda + \mu \\ x_2 = \lambda + \mu \\ x_3 = 2 + \lambda \\ x_4 = -1 + \mu \end{cases} \to \left(\begin{array}{cc|c} 1 & 1 & x_1 - 1 \\ 1 & 1 & x_2 \\ 1 & 0 & x_3 - 2 \\ 0 & 1 & x_4 + 1 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & x_1 - 1 \\ 0 & 0 & x_2 - x_1 + 1 \\ 1 & 0 & x_3 - 2 \\ 0 & 1 & x_4 + 1 \end{array}\right) \to \left(\begin{array}{cc|c} 0 & 0 & x_1 - 1 - x_3 + 2 - x_4 - 1 \\ 0 & 0 & x_2 - x_1 + 1 \\ 1 & 0 & x_3 - 2 \\ 0 & 1 & x_4 + 1 \end{array}\right)$$

$$\Longrightarrow \begin{cases} x_1 - x_3 - x_4 = 0 \\ x_1 - x_2 = 1 \end{cases} \longrightarrow \begin{cases} x_1' + 1 - x_2' - x_3' - x_3' - x_4' = 0 \\ x_1' + 1 - x_1' - x_2' + 1 = 1 \end{cases} \Longrightarrow \pi' \equiv \begin{cases} x_1' - x_2' - 2x_3' - x_4' = -1 \\ -x_2' = -1 \end{cases}$$

(18) $\{\underbrace{(1,-1,2)}_{A}, \underbrace{(2,0,2)}_{B}, \underbrace{(2,-2,2)}_{C}, \underbrace{(1,-1,3)}_{D}\}$ $\quad$ $\{\underbrace{(0,1,1)}_{A'}, \underbrace{(1,1,2)}_{B'}, \underbrace{(1,1,0)}_{C'}, \underbrace{(0,2,1)}_{D'}\}$

$P=(0,1,1)$

Para demostrar que los conjuntos de puntos son referencias afines tenemos que formar 3 vectores $(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD})$ y verificar que son l.i

$\overrightarrow{AB} = (2,0,2)-(1,-1,2) = (1,1,0)$

$\overrightarrow{AC} = (2,-2,2)-(1,-1,2) = (1,-1,0)$

$\overrightarrow{AD} = (1,-1,3)-(1,-1,2) = (0,0,1)$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Longrightarrow rg = 3 \Longrightarrow l.i$$

$R = \{A; \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\} = \{(1,-1,2); (1,1,0), (1,-1,0), (0,0,1)\}$

$\overrightarrow{A'B'} = (1,1,2)-(0,1,1) = (1,0,1)$

$\overrightarrow{A'C'} = (1,1,0)-(0,1,1) = (1,0,-1)$

$\overrightarrow{A'D'} = (0,2,1)-(0,1,1) = (0,1,0)$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \to l.i$$

$R' = \{A'; \overrightarrow{A'B'}, \overrightarrow{A'C'}, \overrightarrow{A'D'}\} = \{(0,1,1); (1,0,1), (1,0,-1), (0,1,0)\}$

$$C(R,R') = C(R_c,R') \cdot C(R,R_c) = C(R',R_c)^{-1} \cdot C(R,R_c) = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)^{-1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Coordenadas del punto $P=(0,1,1)$ resp R

$$\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{cases} 0 = x+y+1 \\ 1 = x-y-1 \\ 1 = z+2 \end{cases} \to \begin{cases} x = 1/2 \\ y = -3/2 \\ z = -1 \end{cases} \quad (1/2, -3/2, -1)$$

Coordenadas del punto P respecto R'

$$\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{cases} 0 = x+y \\ 1 = z+1 \\ 1 = x-y+1 \end{cases} \to \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \quad (0,0,0)$$

(19) $\pi : 5x+3y+z=4 \qquad P=(-2,1,11) \qquad R_a = \{\underbrace{(1,-1,2)}_{A}, \underbrace{(0,1,1)}_{B}, \underbrace{(1,0,-1)}_{C}\}$

Vamos a comprobar que los vectores $\overrightarrow{AB}$ y $\overrightarrow{AC}$ son l.i

$\overrightarrow{AB} = (0,1,1)-(1,-1,2) = (-1,2,-1)$

$\overrightarrow{AC} = (1,0,-1)-(1,-1,2) = (0,1,-3)$

$$\begin{pmatrix} -1 & 2 & -1 \\ 0 & 1 & -3 \end{pmatrix} \to rg = 2 \Longrightarrow l.i$$

$R = \{A; \overrightarrow{AB}, \overrightarrow{AC}\} = \{(1,-1,2); (-1,2,-1), (0,1,-3)\}$

$P = A + \lambda\overrightarrow{AB} + \mu\overrightarrow{AC}$

$$\begin{cases} -2 = 1-\lambda \\ 1 = -1+2\lambda+\mu \\ 11 = 2-\lambda-3\mu \end{cases} \to \begin{cases} \lambda = 3 \\ \mu = -4 \end{cases} \Longrightarrow P' = (1-\lambda-\mu, \lambda, \mu) = (2,3,-4)$$

(20) $S: \begin{cases} x+y-z+t-2r=1 \\ 2x+z-3t=-2 \\ x-y+2z-4t+2r=-3 \end{cases}$ $P=(0,3,1,1,1)$ 2.5

Para hallar una referencia afín de S primero tenemos que dimensión tiene S para saber cuántos puntos necesito en la referencia afín

$$\operatorname{Dim} S = 5 - r_g \begin{pmatrix} 1 & 1 & -1 & 1 & -2 \\ 2 & 0 & 1 & -3 & 0 \\ 1 & -1 & 2 & -4 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & -1 & 1 & -2 \\ 0 & -2 & 3 & -5 & 4 \\ 0 & -2 & 3 & -5 & 4 \end{pmatrix} \Rightarrow \operatorname{Dim} S = 5-2=3$$

Necesito 4 puntos afínmente independientes de $S \rightarrow R_a = \{A, B, C, D\}$

Resuelvo el sistema y consigo 1 punto y 3 vectores

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -2 & 1 \\ 2 & 0 & 1 & -3 & 0 & -2 \\ 1 & -1 & 2 & -4 & 2 & -3 \end{array}\right) \longrightarrow \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -2 & 1 \\ 0 & -2 & 3 & -5 & 4 & -4 \\ 0 & -2 & 3 & -5 & 4 & -4 \end{array}\right) \longrightarrow \left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & -2 & 1 \\ 0 & -2 & 3 & -5 & 4 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow$$

$$\rightarrow \begin{cases} x = \frac{-3\lambda+5\mu-4\gamma+4}{2} - \lambda + \mu - 2\gamma + 1 \\ y = \frac{-3\lambda+5\mu-4\gamma+4}{2} \\ z = \lambda \\ t = \mu \\ r = \gamma \end{cases} \rightarrow \begin{array}{l} P_0 = (3,2,0,0,0) \\ \vec{v}_1 = (-5/2, -3/2, 1, 0, 0) \quad \vec{v}_3 = (-4,-2,0,0,1) \\ \vec{v}_2 = (7/2, 5/2, 0, 1, 0) \end{array}$$

$R = \{(3,2,0,0,0); (-5,-3,2,0,0), (7,5,0,2,0), (-4,-2,0,0,1)\}$ ref. cartesiana

$R_a = \{P_0, P_0+\vec{v}_1, P_0+\vec{v}_2, P_0+\vec{v}_3\} = \{(3,2,0,0,0), (-2,-1,2,0,0), (10,7,0,2,0), (-1,0,0,0,1)\}$

14 $S: \begin{cases} x_1 - x_2 + x_3 - 5x_4 = 1 \\ x_2 + x_3 = 2 \end{cases}$

Necesitamos cuatro vectores y 1 punto $R = \{P_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ $\vec{v}_3, \vec{v}_4 \in \vec{S}$

$\left.\begin{array}{l} \vec{v}_3 = (2, 1, -1, 0) \\ \vec{v}_4 = (5, 0, 0, 1) \end{array}\right\} \in S$ $\quad \left.\begin{array}{l} \vec{v}_1 = (1, 0, 0, 0) \\ \vec{v}_2 = (0, 0, 1, 0) \end{array}\right\} \in A^4$ $\quad P_0 = (1, 1, 1, 0)$

Vamos a comprobar $R = \{(1,1,1,0); (1,0,0,0), (0,0,1,0), (2,1,-1,0), (5,0,0,1)\}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ 1 \end{pmatrix} = \left(\begin{array}{cccc|c} 1 & 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x_1 = x_1' + 2x_3' + 5x_4' + 1 \\ x_2 = x_3' + 1 \\ x_3 = x_2' - x_3' + 1 \\ x_4 = x_4' \end{cases}$$

$$S': \begin{cases} x_1' + 2x_3' + 5x_4' + 1 - x_3' - 1 + x_2' - x_3' + 1 - 5x_4' = 1 \\ \cancel{x_3'} + 1 + x_2' - \cancel{x_3'} + 1 = 2 \end{cases}$$

$$\downarrow$$

$$\begin{cases} x_1' + x_2' = 0 \\ x_2' = 0 \end{cases} \longrightarrow \boxed{\begin{array}{l} x_1' = 0 \\ x_2' = 0 \end{array}}$$

15 $R = \{(1,0,0); (0,0,2), (-1,1,1), (-1,0,1)\}$

Para demostrar que R es un sistema de referencia cartesiano comprobamos si los tres vectores son l.i

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \longrightarrow \text{Si son l.i} \Rightarrow R \text{ es sist. de referencia de } A^3$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right) \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x = -y' - z' + 1 \\ y = y' \\ z = 2x' + y' + z' \end{cases}$$

$$P = (1,1,1) \longrightarrow \begin{cases} 1 = -y' - z' + 1 \\ 1 = y' \\ 1 = 2x' + y' + z' \end{cases} \longrightarrow \boxed{y' = 1} \quad \boxed{z' = -1} \quad x' = \frac{1-1+1}{2};\ \boxed{x' = \frac{1}{2}} \qquad P' = (1/2, 1, -1)$$

Primero vamos a calcular las ecuaciones del subespacio en el sistema de referencia canónico

$$\begin{cases} x = 1 + \mu \\ y = -1 + \lambda \\ z = 1 + \mu \end{cases} \longrightarrow \left(\begin{array}{cc|c} 0 & 1 & x-1 \\ 1 & 0 & y+1 \\ 0 & 1 & z-1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 0 & 1 & x-1 \\ 1 & 0 & y+1 \\ 0 & 0 & z-1-x+1 \end{array}\right) \longrightarrow z - x = 0$$

$$2x' + y' + z' + y' + z' - 1 = 0$$

$$\boxed{2x' + 2y' + 2z' = 1}$$

BLOQUE II. SISTEMAS DE REFERENCIA

(11) $R = \{(2,1);\ (2,4),(-1,3)\}$ $\quad P = (4,5)$ $\quad r: y = 2x+1$

Primero vamos a pasar del sistema de referencia canónico el sistema de referencia R que nos dan

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \left(\begin{array}{cc|c} 2 & -1 & 2 \\ 4 & 3 & 1 \\ \hline 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x = 2x' - y' + 2 \\ y = 4x' + 3y' + 1 \end{cases}$$

$$P = (4,5) \longrightarrow \left.\begin{array}{l} 4 = 2x' - y' + 2 \\ 5 = 4x' + 3y' + 1 \end{array}\right\} \longrightarrow y' = 2x' - 2$$

$$4x' + 3(2x'-2) + 1 - 5 = 0$$

P en $R = (1,0)$

$$10x' = 10$$

$$x' = 1, \quad y' = 0$$

$$r: y = 2x+1 \longrightarrow 4x' + 3y' + 1 = 2(2x' - y' + 2) + 1$$

$$4x' + 3y' + 1 = 4x' - 2y' + 4 + 1$$

$$5y' = 4, \quad \boxed{y' = 4/5}$$

(12) $R = \{(-1,0,0);\ (1,1,0),(0,-1,0),(0,0,-1)\}$

Pasemos de R_c a R:

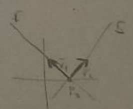
$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \longrightarrow \begin{cases} x = x' - 1 \\ y = x' - y' \\ z = -z' \end{cases}$$

a) $\pi: 2x - y + z + 2 = 0 \longrightarrow 2(x'-1) - (x'-y') + (-z') + 2 = 0$

$$\boxed{x' + y' - z' = 0} \longrightarrow \begin{cases} x' = -\lambda + \mu \\ y' = \mu \\ z' = \lambda \end{cases}$$

b) $r: \begin{cases} 2x + y = 0 \\ x - 2y + z = 1 \end{cases} \longrightarrow \begin{cases} 2(x'-1) + (x'-y') = 0 \\ x' - 1 - 2(x'-y') - z' = 1 \end{cases} \Longrightarrow \begin{cases} 3x' - y' = 2 \\ -x' + 2y' - z' = 2 \end{cases}$

(13) $r: x + y = 1$ $\quad s: x - y = 1$



Necesitamos un punto y dos vectores $R = \{P_0;\ \vec{v}_1, \vec{v}_2\}$

$\vec{v}_1 = (1,-1)$ $\quad \vec{v}_2 = (1,1)$ $\quad P_0 = (1,0)$ punto intersección de las dos rectas

Comprobamos que este sist. de referencia es el correcto

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & 1 & 0 \\ \hline 1 & 1 & 0 \end{array}\right) \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \longrightarrow \begin{array}{l} x = x' + y' + 1 \\ y = -x' + y' \end{array} \longrightarrow$$

$r': x' + y' + 1 - x' + y' = 1$ $\quad \boxed{y' = 0}$

$s': x' + y' + 1 + x' - y' = 1$ $\quad \boxed{x' = 0}$